25 - 26 KTMC Maths Club Presents

Math Magazine

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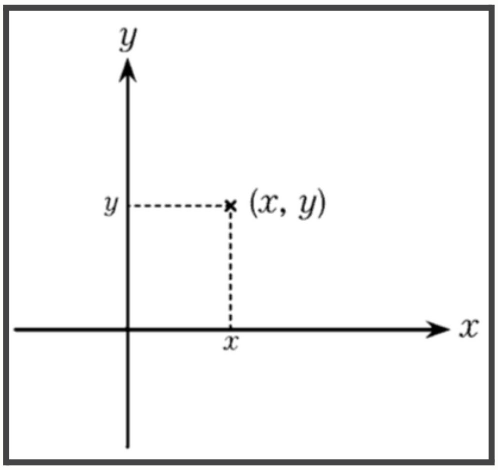
THE FLY ON THE CEILING (Coordinate Geometry)

The famous 17th-century French mathematician **René Descartes** was physically weak from childhood. Because of this, the school specially allowed him to stay in bed to rest every morning while his classmates were attending morning prayers. For Descartes, this quiet morning time was the golden hour for him to contemplate scientific and philosophical questions.

(René Descartes)

Legend has it that on an ordinary morning, Descartes was lying in bed, staring at the ceiling. He noticed a fly buzzing around. Suddenly, an interesting thought popped into his head: "How can I accurately record the specific position of this fly at any given moment?" After a period of observation, he had a stroke of genius: If one corner of the ceiling is treated as the **starting point (the "Origin")**, then the fly's position can be perfectly locked down by measuring its **horizontal distance (x)** from one wall and its **vertical distance (y)** from the adjacent wall!



Precisely because Descartes watched the fly on the ceiling, he realized that we can use algebraic equations to solve geometric problems! This great discovery successfully combined the two fields, announcing the birth of **"Coordinate Geometry"**.

> ★ *By calculating the **intersection points of circular signal orbits** emitted by satellites or base stations, GPS can use **triangulation** to accurately pin down your location (which utilizes concepts of **coordinate geometry**).*

THE BIRTHDAY PARADOX (Probability)

Imagine this: in a secondary school class of 23 students, do you think the chance that "**at least two classmates share the same birthday**" is high?

Assuming the **probability of being born on any day is equal** and **ignoring leap years**, we can use probability theory to derive this question from the "opposite side"—that is, calculating the probability that "**everyone's birthday is completely different**"

Could it possibly exceed 50%?

Most people think that since a year has 365 days, how could it be such a coincidence for a mere 23 people?

The answer is: 50.7%

The probability of the 1st student having a unique birthday is: $\frac{365}{365} = 1$

The 2nd student's birthday cannot repeat the 1st one's, so the probability is: $\frac{364}{365}$

Following this pattern up to the 23rd student, the probability is: $\frac{343}{365}$

Multiplying the probabilities of these 23 people together:

Probability that everyone has a different birthday

$$= \frac{365\times364\times363\times\ldots\times343}{365^{23}}$$

$$\approx 49.3\%$$

The probability that **at least two people share the same birthday** is:

$$100\% - 49.3\% = 50.7\%$$

Taylor Series (Taylor’s Approximation)

When calculating the probability of the "Birthday Paradox," as the number of people n gradually increases, directly calculating $\frac{365 \times 364 \times \ldots \times (365-n+1)}{365^n}$ becomes extremely tedious. To simplify the calculation, mathematicians usually introduce **Taylor's Approximation**.

（Brook Taylor）

In a class of n people, the probability that **everyone's birthday is completely different** is:

$$P(different) = \prod_{k=1}^{n-1} \left(1 - \frac{k}{365}\right)$$

➔ $\prod$ is the product symbol

According to the Taylor series in calculus, the expansion of the exponential function e^{-x} near $x = 0$ is：

Formula：

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots$$

When the value of x is very small（$x \approx 0$）, values like x^2, x^3 become **relatively minuscule** and **can be ignored**. Therefore, we obtain a classic **linear approximation formula**: $1 - x \approx e^{-x}$ 。

Since a year has 365 days, when k is relatively small, $\frac{k}{365}$ is a very small fraction. At this point, we can substitute x with $\frac{k}{365}$ and apply the Taylor approximation:

$$\left(1 - \frac{k}{365}\right) \approx e^{-\frac{k}{365}}$$

Now, substituting this approximation back into the original product formula:

$$P(different) \approx e^{-\frac{1}{365}} \times e^{-\frac{2}{365}} \times \ldots \times e^{-\frac{n-1}{365}}$$

Taylor Series (Taylor's Approximation)

According to the **Law of Indices** and **Arithmetic Progression** :

Formula :

$$a^m \times a^n = a^{m+n}$$

Formula :

$$S_n = \frac{n(a_1 + a_n)}{2}$$

We get :

$$P(different) \approx e^{-\frac{n(n-1)}{730}}$$

Therefore, the probability that **at least two people share the same birthday** is:

$$P(same) \approx 1 - e^{-\frac{n(n-1)}{730}}$$

Let us use $n = 23$ to test the approximation.

$$P(same) \approx 1 - e^{-\frac{23(23-1)}{730}}$$

$$\approx 50\%$$

The exact calculated answer is **50.7%**, while the answer using Taylor's Approximation is **50.0%**. The two differ by a mere **0.7%**!

THE MOST BEAUTIFUL FORMULA (Euler's identity)

In the mathematical world, there is a formula universally recognized as the "ultimate paradigm of mathematical beauty": **Euler's identity**.

Formula: $e^{i\pi} + 1 = 0$

Named after the famous Swiss mathematician **Leonhard Euler**, this formula astounds countless scientists because it perfectly blends the **five fundamental constants** of mathematics together:

(Leonhard Euler)

0: **Additive identity**
The identity element for addition (any number plus 0 remains unchanged).
1: **Multiplicative identity**
The identity element for multiplication (any number multiplied by 1 remains unchanged).
π: **Pi** ≈ 3.14159...
The ratio of a circle's circumference to its diameter, which is the core of geometry.
e: **Euler's number** ≈ 2.71828...
The base of natural logarithms, widely used in calculus and mathematical analysis.
i: **Imaginary unit**
Defined as $i^2 = -1$, the cornerstone of the complex number world.

In fact, Euler's identity is a special case of Euler's formula $e^{ix} = \cos x + i \sin x$.

When we substitute x as π, since $\cos\pi = -1$ and $\sin\pi = 0$, the formula simplifies to $e^{i\pi} = -1$. Moving the term over gives us this stunning equation.

> ★ *By converting the exponential function $e^{i\theta}$ into sine and cosine waves, Euler's formula connects complex mathematical numbers to periodic vibration phenomena in the real world, such as the swinging of a pendulum, sound waves, and ocean waves.*

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KTMC 25-26 Maths Club

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